Comparative Study of Jet-Quenching Schemes

Working towards a unified approach in Jet-modification

A. Majumder, Duke University

OUTLINE

- A brief history & time-line
- Four schemes in four dimensions!
- A comparison of extremes
- A meeting ground (finding agreement)
- Going beyond $R_{AA}$, Space-time Profiles.
- Phenomenological extensions (jet correlations)
- Comprehending the landscape @ RHIC
Classify using scales in the problem

$Q^2$

$\mu^2$

$E >> Q, \mu$

$x = E_g/E$

$\hat{q} = \frac{\sum_i \mu_i^2}{L}$

- Difference between schemes $\rightarrow$ relation between $Q$ and $\mu$
- Extending schemes beyond pQCD with models
Higher Twist Approach, $E \gg Q \gg \mu$

- A medium with a color correlation length $\lambda \ll L$
- Highly VIRTUAL parton produced in hard collision
- Parton picks up FEW SMALL transverse kicks $\sim \mu^2$
- Expand diagrams in $\mu/Q$, transverse kick in $k_T A^+ \rightarrow F_T^+$
- Formal similarity with DGLAP evolution $0<x<1$.
- Interference between diagrams leads to LPM suppression

• Entire effort is calculating the modification of $D(z)$

$$\frac{d \sigma}{dy dp_T} \sim \int dx_a dx_b G(x_a) G(x_b) \frac{d \hat{\sigma}}{d \hat{t}} D_h^q(z_1)$$

• E-Loss estimate or gluon radiation intensity not needed to get spectra
• Extract length enhanced contributions $(\mu^2/Q^2) L$
• Modification looks like DGLAP $X$ factor

The main difference is in the fragmentation functions

$$D^{AA} = D + \Delta D (\text{medium})$$

$$\Delta D \sim - \int d l_T^2 d z f(l_T, R_{\text{origin}}, R_{\text{length}}, x_B) P(z) \times \int d y_1^- d y_2^- \langle \text{Matter} | G^{+\mu}(y_1^-) G_{+\mu}(y_2^-) | \text{Matter} \rangle$$
GLV, Recursive Operator in Opacity

\[ E \gg Q \sim \mu \]

- Medium of heavy (static) scattering centers with Yukawa like potentials
- Parton picks up transverse kicks \( \sim \mu^2 \)
- Operator formalism that sums order by order in opacity
- Approximate gluon \( x \to 0 \) (soft gluons), ignore spins!
- Interference between different diagrams leads to the LPM effect and the \( L^2 \) length dependence of E-loss.

\[ n = \frac{L}{\lambda_g} \]

• Central quantity: radiated gluon intensity, per emission!
• Gives direct measure of E-loss
• Jet loses energy in multiple tries
• To get Hadrons, need to get distribution of E-loss: $P(E)$

$$P(E) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \prod_{i=1}^{n} \int d\omega_i \frac{dN(\omega_i)}{d\omega} \right] \times e^{-\int d\omega \frac{dN}{d\omega}} \delta\left( E - \sum_{i=1}^{n} \frac{\omega_i}{E_{jet}} \right)$$

• Used to calculate the energy shifted fragmentation function

$$D_{h/q}^{med}(z, Q^2) = \int_{0}^{1} d\varepsilon \ P(\varepsilon) \frac{1}{1 - \varepsilon} D_{h/q}^{vac} \left( \frac{z}{1 - \varepsilon}, Q^2 \right)$$
ASW, Path integral in Opacity

\( E \gg \mu \sim Q \)

- Medium of heavy (static) scattering centers with Yukawa potentials
- Parton picks up perp. momentum from kicks in medium
- Path-integral in opacity
- Two simple limits of calculation
  a) Few hard scatterings (GLV)
  b) Many soft scatterings (BDMPS)

\[
\frac{-n}{\lambda_g} = \frac{L}{\lambda_g}
\]

• Central quantity: radiated gluon intensity, per emission

• Gives direct measure of E-loss

• To get Hadrons, need to get distribution of E-loss: $P(E)$ and use that to get a fragmentation function

$$P_E(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \prod_{i=1}^{n} \int d\omega_i \frac{dI(\omega_i)}{d\omega} \right] \delta \left( \epsilon - \sum_{i=1}^{n} \frac{\omega_i}{E} \right) \exp \left[ - \int d\omega \frac{dI}{d\omega} \right]$$

$$P(\Delta E) = P_0 \delta(\Delta E) + P(\Delta E)$$

$$D_{h/q}^{(med)}(z, Q^2) = \int_0^1 d\epsilon P_E(\epsilon) \frac{1}{1-\epsilon} D_{h/q} \left( \frac{z}{1-\epsilon}, Q^2 \right).$$

$$\frac{dI}{d\omega} = \int_0^\omega dk_\perp \omega \frac{dI}{d\omega dk_\perp}$$

BDMPS $(R=\infty)$

$R=40000$

$R=10000$

$R=1000$

$R = \frac{1}{2} \hat{q}L^3, \quad \omega_c = \frac{1}{2} \hat{q}L^2$

BDMPS $\Rightarrow R \rightarrow \infty$
AMY-Finite temperature field theory approach, $E \gg \mu \gg Q$

- Hot thermal medium of quarks and gluons at $T \to \infty$
- $T \to \infty$ implies $g \to 0$
- Hard parton comes in onshell $E \sim T$
- Picks up multiple soft hits, $\mu \sim gT$ from hard particles of $\sim T$
- The hard lines never go off-shell by more than $g^2T$
- Long formation time leads to multiple scattering

• One calculates the cuts of infinite series of ladder diagrams,

• Gives rates of change of quark and gluon distributions

\[
\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k) \frac{d\Gamma_{q\bar{q}}^q(p+k, k)}{dk dt} - P_{q\bar{q}}(p) \frac{d\Gamma_{q\bar{q}}^q(p, k)}{dk dt} + 2P_g(p+k) \frac{d\Gamma_{q\bar{q}}^g(p+k, k)}{dk dt},
\]

\[
\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p+k) \frac{d\Gamma_{q\bar{q}}^g(p+k, p)}{dk dt} + P_g(p+k) \frac{d\Gamma_{q\bar{q}}^g(p+k, k)}{dk dt} - P_g(p) \left( \frac{d\Gamma_{q\bar{q}}^g(p, k)}{dk dt} + \frac{d\Gamma_{q\bar{q}}^g(p, k)}{dk dt} \Theta(2k-p) \right),
\]

• The shifted distributions are used to get the medium modified fragmentation function

\[
D_{\pi,c}(z, Q; r, n) = \int dp_f \frac{z'}{z} \left( P_{q\bar{q}/c} (p_f, p_i) D_{\pi/q} (z', Q) + P_{g/c} (p_f, p_i) D_{\pi/q} (z', Q) \right)
\]
Comparing Results for $R_{AA}$:

**GLV** vs **ASW**

$q = 5-15 \text{GeV}^2/fm$

$\hat{q} = 1 - 2.5 \text{ GeV} \cdot f m^{-1}$
Comparing Results for $R_{AA}$:

HT vs AMY

- $q\cdot\text{max} = 3-4 \text{ GeV}^2/\text{fm}$
- $\langle q\cdot\hat{\text{h}} \rangle \sim 1 \text{ GeV}^2/\text{fm}$

- $\langle q\cdot\hat{\text{h}} \rangle \sim 2 \text{GeV}^2/\text{fm}$ at $T = 370 \text{ MeV}$
The model landscape!

Assumptions about the medium

Finite temp. resummation, (AMY)
- Equilibrated infinite medium, HTL dispersion relations
- Infinite number of scatterings resummed
- $T \to \infty$ makes $\alpha_s$ small

Recursive Operator Opacity, (GLV)
- Medium made of heavy scattering centers
- Finite number of scatterings
- Large $Q \mu$ makes $\alpha_s$ small

Path Integral Opacity, (ASW)
- Medium made of heavy scattering centers
- Infinite number of scatterings resummed

Similar models

Different models

Different q-hat

scale $\sim T \leftarrow \alpha_s$ small $\rightarrow$ Jet scale $\sim Q$
AMY based on Hard Thermal Loops: An effective theory of soft modes in a very hot plasma, (Braaten, Pisarski 1990)

Alternate picture, A Vlasov theory of hard particles in soft fields, (Blaizot, Iancu 1992)
Medium enhanced higher twist isolates near on-shell propagation in large nuclei

Mean transverse momentum shift

\[ k_T^2 = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} L \int ds^- \left< F^{+,a}_\mu (s^-) F^{+,a}_\mu (0) \right> \]


Propagation of a colored particle in a color field with a short distance correlation,

Langevin Eqn with a lorentz force.

\[ \frac{dk_T(t)}{dt} = g Q^a (E^a + \nu \times B^a) \]

Fields have short correlation lengths

Mean transverse momentum shift

\[ k_T^2 = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} L \int ds^- \left< F^{+,a}_\mu (s^-) F^{+,a}_\mu (0) \right> \]
But radiation vertex is slightly different

H-T has two kinds of contributions

Original parton comes out of hard scattering off shell

Off shell-ness generated later by multiple scattering

And interferences

First set of diagrams missing in AMY, but has multiple scattering contributions to second type

How important is this difference?

will need to look at differential observables
Differentiating between Model differences with differential observables (More data)!

- Jet-medium and multi-particle correlations
- Settle questions of scheme
- Phenomenological extensions to lower momenta
- Use jets to probe bulk dynamics
- And Microscopic degrees of freedom
- Understanding q-hat is understanding the medium.
- How does the medium:
  stop jets, turn jets, distribute lost energy
Q-hat depends on space-time profile of density

- Set a q-hat maximum
- Modulate with space-time profile
- Azimuthally dependent $R_{AA}$ can distinguish

A. Majumder, nucl-th/0608043
• Flow can confuse the difference

• Need many observables in tandem to set the gluon density

• Like $R_{AA}$ vs centrality and back-to-back dihadrons

Central events can give the same answer by adjusting $q_{*}$ but actual dynamics is different, Renk!, see talk in 1.3

Region is very model-dependent, but jets penetrate the core $\rightarrow$ expansion!
Q-hat is a tensor

Non-Isotropic medium $\rightarrow$ non-isotropic q-hat

- Imagine large turbulent magnetic fields produced early in plasma
- B fields are transverse to the beam
- Will deflect jets, preferentially out to large rapidities
- Near side correlations in $\eta$ effected !!
- Q-hat not just from entropy carrying degrees of freedom

$$\hat{q}_{\alpha\beta} = \frac{\kappa_{T\alpha} \kappa_{T\beta}}{L}$$

See talk by M. Strickland
Lead to the formation of an extended ridge on the near side

- Note: broadening only in eta and not in phi
- Introducing a transverse momentum into the Yukawa potential or directional q-hat – ASW
- GLV- no broadening, only shift !!
How does lost energy show up in bulk matter

Jet correlations

- Correlate a hard hadron with another one
- Hard-Hard correlations understood within p-QCD, Consistency check!

Same side

Away side

Majumder

Renk
High $p_T$ on away side, differential

Also understood within pure jet-quenching schemes,
Higher twist calculations, NLO hard part, Full geometry

See talk by H. Z. Zhang, Parallel 2.2
Low pT on the away side,
Mach-Cherenkov-Gluon Brem. cone

- More theory that you would ever want!

- Including flow is better for Mach cone explanation

Cassalderry-Solana

Renk & Ruppert
Alternate explanations still persist and have not been ruled out

- Cherenkov radiation,
- V. Koch, AM, X-N. Wang

Regular Gluon radiation with Sudakov form factor for no emissions!
A. Polosa, C. Salgado

- Not a two state problem, but more complicated!

- Not a deflected jet! From 3 particle correlations (talk by C. Pruneau), progress..
Conclusions

- Jet-modification in dense matter  
  (Well motivated and unsettled)
- Different schemes, using similar physics
- Differences in implementation (must be resolved!!)
- Multi-particle correlations to high $p_T$
- Model extensions at lower $p_T$.
- Explore the space-time profile of the medium
- Explore the many dimensions of q-hat
- Need more data, need more statistics!

Topics left out: Heavy-quarks, elastic energy loss,
Back up!
Comparing different ST profiles at most central events

- Preliminary
E loss formulae!

\[ \Delta E_{ASW} = \frac{\alpha_s C_R}{4} n_0 \mu^2 L^2 \log \left( \frac{E}{\mu} \right) = \Delta E_{GLV} \]

\[ \Delta E^{(1)} \approx \frac{C_R \alpha_s}{4} \frac{\mu^2 L^2}{\lambda_g} \log \frac{2E}{\mu^2 (L)L} \]

\[ \Delta E \approx \frac{\alpha_s}{\pi} N_c \frac{m_D^2}{\lambda} L^2 \]
What is $D_{AA}$

\[ D_{AA}(z_T) \equiv p_T^{\text{trig}} \frac{d\sigma / dp_T^{\text{asso}} dp_T^{\text{trig}}}{d\sigma / dp_T^{\text{trig}}} \]

\[ = \int dE_T \frac{d\sigma}{dE_T} D\left(\frac{p_T^{\text{trig}}}{E_T}\right) D\left(\frac{p_T^{\text{asso}}}{E_T}\right) \]

\[ = \int dE_T \frac{d\sigma}{dE_T} D\left(\frac{p_T^{\text{trig}}}{E_T}\right) \]

\[ z_T = \frac{p}{p_{\text{trig}}} \]
The results! Simplest thing $R_{AA}$

\[ \pi^0 \text{ 0-10\% Central } \]
\[ Au+Au \sqrt{s_{NN}} = 200\text{GeV} \]

- PHENIX preliminary

\[ \pi^0 \text{ 0-5\% Central } \]
\[ Au+Au \sqrt{s_{NN}} = 200\text{GeV} \]

- PHENIX preliminary

- Dainese, talk at PANIC05

\[ R_{AA} \]

\[ p (\text{GeV/c}) \]

\[ R_{AA} \]

\[ p (\text{GeV/c}) \]
How can such fundamentally different physics produce equally good description of data???,

B. Cole QM2005, problem weirder than you think!

• GLV and ASW are very similar in basic structure (different qhat)
• Difference in implementation, Geometry!
• AMY and HT have truly different origins (similar qhat)
• How can they yield similar physics ???
• Deeper similarity between AMY and HT.
• Within the approximation schemes, they may be similar physics
• Look at Basic theory without radiation in HT and AMY